

# **Improved Analysis of Unbalanced Feistel Networks by Coupling**

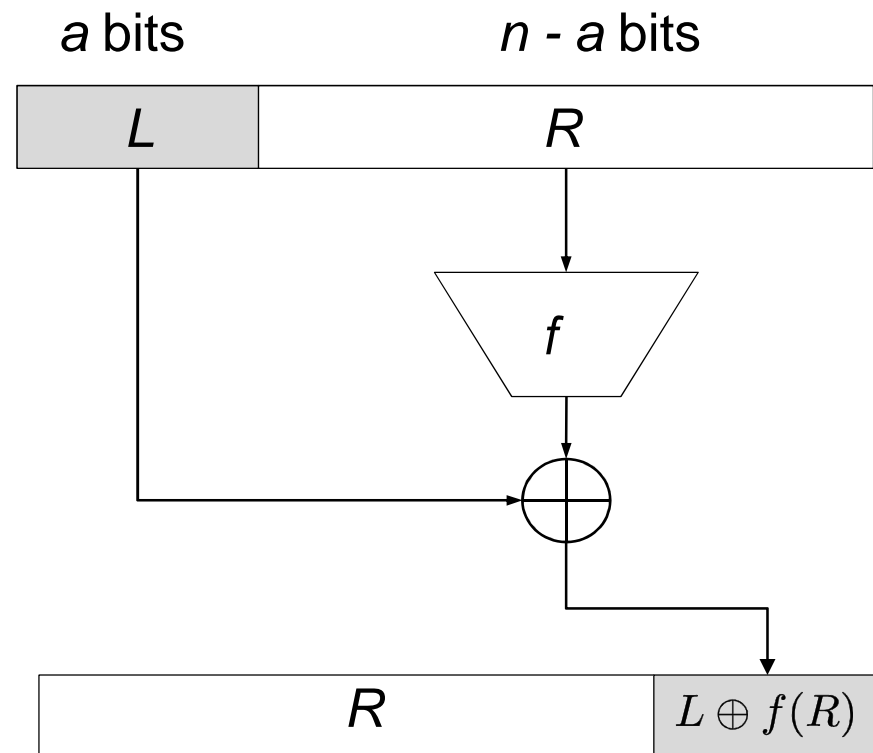
Viet Tung Hoang & Phillip Rogaway

University of California, Davis

# Unbalanced Feistel networks

- By [Schneier-Kelsey]

Feistel  $(n, a)$ -network



# CCA-security of unbalanced Feistel networks

$a$	Queries
$a$	about $2^{(n-a)/2}$ [Naor - Reingold]
$n/2$	about $2^{n/2}$ [Patarin]
1	about $2^{n-1}$ [Morris et al.]

# Our results

- **Theorem:** If a Feistel  $(n, a)$ -network  $E$  has  $\frac{4rn}{a}$  rounds then

$$\mathbf{Adv}^{cca}(E, q) \leq \frac{q}{r+1} \left( \frac{2nq}{a} \cdot 2^{a-n} \right)$$

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- Simple proof by **coupling argument**.

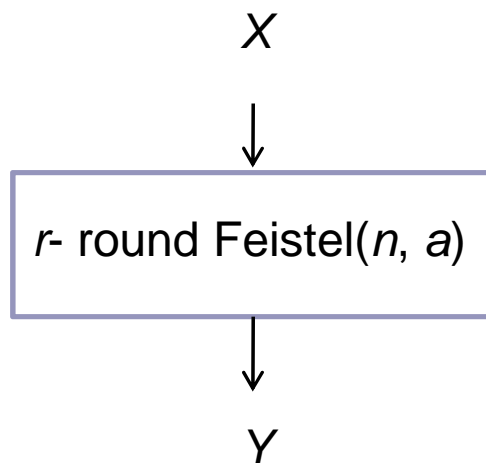
# Intuition of the proof

- Adversary asks nCPA queries  $X_1, \dots, X_q$



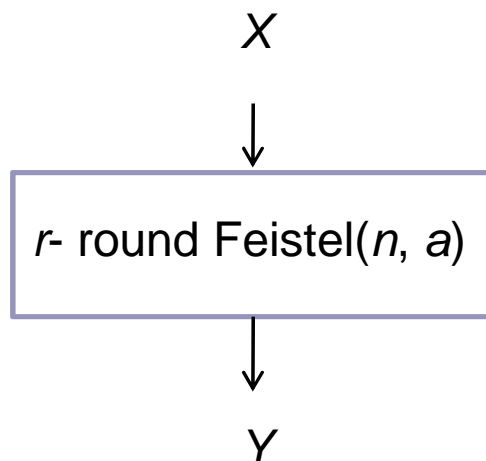
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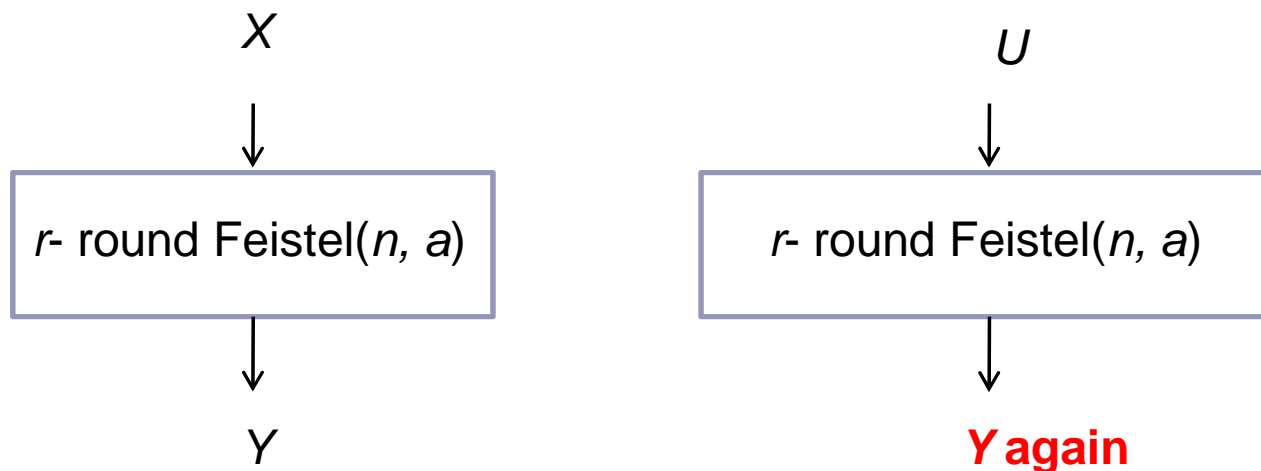
# Intuition of the proof

- Adversary asks nCPA queries  $X_1, \dots, X_q$
- $X = (X_1, \dots, X_q)$ , and  $Y$ : the vector of outputs from  $X$
- $\pi$ : uniform distribution in the set of  $q$ -tuples of elements of  $\{0, 1\}^n$



# Intuition of the proof

- Pick random vector  $U = (U_1, \dots, U_q)$  with distribution  $\pi$
- Design a new Feistel  $(n, a)$ -network that outputs  $Y$  on input  $U$ , with high probability.



# Intuition of the proof

